REPORT DOCUMENTATION PAGE	*Form Approved OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
31-07-2012	FINAL	1 May 2009 - 30 April 2012

4. TITLE AND SUBTITLE
Mean Field Control of Large Population Stochastic Systems

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9950-09-1-0323

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Peter E. Caines

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
The Royal Institution for the Advancement of Learning
3480 University Street, Montreal, Canada QCH3A-2A7

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

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Arlington VA 22203
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10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-OSR-VA-TR-2012-1085

12. DISTRIBUTION/AVAILABILITY STATEMENT
DISTRIBUTION A: APPROVED PUBLIC RLEEASE

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Multi-agent competitive and cooperative systems occur in a vast range of designed and natural settings such as communication, economic, energy and transportation systems. However, the complexity of such large population stochastic dynamic systems, and frequently their inherent nature, make centralized control infeasible or irrelevant and standard game theoretic analysis intractable. The key idea of Mean Field (MF) stochastic control (or Nash Certainty Equivalence (NCE) control) is that when the agent population is very large individual feedback strategies exist for all of the agents so that each agent will be in an approximate Nash equilibrium with with the pre-computable behaviour of the mass of other agents. During the period of receipt of this award significant progress in this research area has been made in this subject area. In particular, as planned, the following topics have been analyzed and computational methodologies generated: (i) a fundamental theory and computational methodology of adaptive MF Stochastic systems, (ii) a MF Games theory of Consensus and Flocking Systems, (iii) MF Leader-Follower and Egoist-Altruist syste

15. SUBJECT TERMS
stochastic control, decentralized control, mean field control, game theory

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
U	U	U	U		Peter E. Caines

19b. TELEPHONE NUMBER *(Include area code)*
1 514 398 7129

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Final Report July 2012

Mean Field Control of Large Population Stochastic Systems

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Introduction

Multi-agent competitive and cooperative systems occur in a vast range of designed and natural settings such as communication, economic, energy and transportation systems. However, the complexity of such large population stochastic dynamic systems, and frequently their inherent nature, make centralized control infeasible or irrelevant and standard game theoretic analysis intractable. The key idea of Mean Field (MF) stochastic control (or Nash Certainty Equivalence (NCE) control) is that when the agent population is very large individual feedback strategies exist for all of the agents so that each agent will be in an approximate Nash equilibrium with with the pre-computable behaviour of the mass of other agents. During the period of receipt of this award significant progress in this research area has been made in this subject area. In particular, as planned, the following topics have been analyzed and computational methodologies generated: (i) a fundamental theory and computational methodology of adaptive MF Stochastic systems, (ii) a MF Games theory of Consensus Systems, (iii) MF Leader-Follower Systems, (iv) applications to power markets, and (v) a fundamental analysis of non-linear Mean Field Systems with Major and Minor players.

In this Final Report the main sections are devoted to the five topics listed above. This comprehensive summary of the significant work accomplished within this research program is followed by bibliography of the publications which have been generated by the PI and his collaborators.

1. Stochastic Adaptive Mean Field Control

The inclusion of learning procedures for the identification by a given agent of the dynamical and cost function parameters of other competing agents in a stochastic dynamic system, or of the statistical distribution of these parameters in a mass of competing agents, introduces new features into the system theoretic NCE (MF) setup.

The natural initial problem in the development of adaptive MF stochastic system theory is that where each agent needs to estimate its own dynamical parameters, while its control actions are permitted to be explicit functions of the parameter distribution of the entire population of competing agents. A subsequent problem is

the generalization where each agent also needs to estimate the distribution of the population's dynamical parameters, and a natural further generalization is the case where the cost function parameters also vary over the population and this distribution is unknown to each agent and hence needs to be estimated. In this paper we provide a solution to the most general problem in this sequence.

The work in this program under this heading provides a study of the mean field stochastic adaptive optimal control problem where the cost functions of the agents in a population are coupled, and each agent estimates its own dynamical parameters based upon observations of its own trajectory, and furthermore estimates the distribution parameter of the population's dynamical and cost function parameters by observing a randomly chosen fraction of the population. This work makes a contribution to the mean field literature by extending the established epsilon-Nash equilibrium results of a large population of egoistic agents to a large population of adaptive egoistic agents. The information requirement for each agent is kept limited in the sense that the distribution of the dynamical parameters of the population is estimated only through a fraction of the population which becomes negligible as the population size grows to infinity. The strong consistency of the self-parameter estimates and the population distribution function parameter estimates, the stability of the system, and an epsilon--Nash Equilibrium property are all established in this analysis.

All of the work on stochastic adaptive MFG control has either been published in journals, presented at conferences or has been accepted for publication (see the list of papers in Subsection 1 of the References section).

2. Mean Field Theory of Consensus and Social Systems

A consensus process is the process of dynamically reaching an agreement between the agents of a group on some common state properties such as position or velocity. The formulation of consensus systems is one of the important issues in the area of multi-agent control and coordination, and has been an active area of research in the systems and control community over the past few years.

In the Mean Field (MF) dynamic game consensus model considered in this work:
(i) each agent has a priori information on the initial state distribution mean of the overall population, (ii) the set of MF control laws possesses an epsilon-Nash equilibrium property, (iii) the system of agents reaches consensus and does not require communication with other agents. Whereas in the Standard Consensus (SC) algorithms: (i) agents need no a priori information on the initial state distribution of the overall population but require local communication with other agents, (ii) consensus can be achieved if the union of the interaction graphs for the system is connected frequently enough as the system evolves. Furthermore, in the deterministic problem formulation, we show that a finite population system with the observation feedback algorithm reaches consensus on the initial state distribution mean as time and population size N go to infinity.

In the deterministic problem formulation for a finite population system, we show that the LRA cost of each individual at the MF Nash equilibrium, the minimal LRA social cost with decentralized MF strategies and the minimal LRA social cost with centralized information are equal to zero. However, the transient solutions of these social optimal strategies will in general be different. The SC algorithms require global communication with other agents (or local communication with neighbors in the random and time-varying network topologies) in the system and for large N this leads to high communication and computational complexity. On the other hand, the decentralized social MF control laws do not require even local communication and hence are robust with respect to communication network failures, but to gain this property a priori information on the mean of the system's initial state distribution must be available to each agent (see the papers and conference items in References Subsection 2). This problem formulation further applies to Cucker-Smale type flocking systems (see References Subsection 2).

Linked to this work is the studies carried out of the emergence of coalitions in MF systems and of the properties of mixed populations of major-minor and egoist-altruist MF games systems, and moreover, the investigations of the relationship between competitive game theoretic MF system behaviour and social cooperative behaviour (again see the papers and conference items in References Subsection 2).

3. Mean Field Leader-Follower Systems

In this set of work a game theory based model of collective dynamics has been produced which includes leaders, followers and a reference trajectory to be tracked. The mean field equations characterizing the Nash equilibrium for infinite population systems were derived, and under appropriate conditions, they have a unique solution leading to decentralized control laws. Furthermore, for large but finite population systems, such controls were shown to correspond to so-called epsilon-Nash equilibria.

The computation of the followers' control laws requires knowledge of the complete reference trajectory of the leaders which is in general not known to the followers. In response to this we have proposed and tested a reference trajectory likelihood ratio based adaptation scheme based on noisy observations by followers of a random sample of leaders. Under appropriate identifiability conditions, it is established that this identification scheme is able to select the exact reference trajectory model within a finite class of candidates in a finite deterministic time almost surely as the number of samples goes to infinity. As a result, the two phase (estimation based) adaptive mean field control laws of the followers together with the mean field control laws of the leaders give rise to a dynamic stochastic Nash equilibrium for the

overall leader-follower system (see the papers and conference items in References Subsection 3.)

4. Mean Field Theory Applications to Power Markets

The term "smart grid" refers to the incorporation of recent advances in communication and computation into the grid; this is in order to increase the connectivity, automation and coordination among these suppliers, consumers and networks which perform transmission or distribution tasks. The long-term goal is to formulate a power system model and associated control laws where highly intermittent suppliers can be accommodated, peak demand can be reduced, and dependency on polluting fossil fuels with their volatile price can be decreased.

One of the important innovations that the smart grid offers is the replacement of analog mechanical meters with smart meters, that is to say digital meters which have fast transmission capabilities that can carry instant information such as the locational marginal price of the grid, and have a certain amount of computation power. Even though technological advances enable the use of smart meters which facilitates the demand response mechanism, there are several issues to be considered before these useful devices can be integrated in the large scale. algorithm where the consumers and suppliers only follow the price signal measured from the smart meters and have statistical information measured from the entire population. We propose a decentralized algorithm that gives the best response action for each agent for an infinite population. The algorithm guarantees an epsilon-Nash equilibrium and stability for the finite population system under the strong assumption that the population dynamical parameter distribution is perfectly known to all agents in the system. The model in this paper is highly stylized in order to obtain analytical tractability, and some of the assumptions are hard to satisfy in a real power market. However, most of these assumptions can be softened by either incorporating numerical analysis techniques to solve the partial differential equations or by data driven analysis in order to capture the instantaneous population parameter distributions, etc.

The price trajectory forecast carries valuable information for consumers, suppliers and the operators. For the suppliers and consumers it gives the opportunity to decide to startup or shutdown its current load or supply, which is a costly action.
For the operators it gives a chance to make an action in case of an oscillating or volatile price trajectory forecast. Mean Field (MF) Stochastic Systems Best response calculation in a dynamic large population game requires complete state observations on the population for each agent. The complexity becomes intractable as the population gets larger. For these problems, the mean field framework

provides decentralized strategies that yield Nash equilibria in the asymptotic limit of an infinite (mass) population. The control laws use only the local information of each agent on its own state and own dynamical parameters, while the mass effect is calculated offline using statistical information. These laws yield approximate equilibria when applied in the finite population (see the Reference Subsection 4).

In this work the mass effect has two components, namely the consumer and supplier masses. These two population partitions have different characteristics; however, in the population limit, the mean field equations provide the smooth deterministic joint mass effect.

5. Non-linear Mean Field Systems with Major and Minor Players

This work studies a stochastic mean field system for a class of dynamic games involving nonlinear stochastic dynamical systems with major and minor (MM) agents. The SMF system consists of coupled (i) backward in time stochastic Hamilton-Jacobi-Bellman equations, and (ii) forward in time stochastic McKean Vlasov or stochastic Fokker-Planck-Kolmogorov equations. Existence and uniqueness of the solution to the MM-SMF system is established by a fixed point argument in the Wasserstein space of random probability measures. In the case that minor agents are coupled to the major agent only through their cost functions, the epsilon-Nash equilibrium property of the SMF best response control possess is shown for a finite N population system where epsilon_N=O(1/N^(1/2)). As a particular but important case, the results of Nguyen and Huang (2011) for MM stochastic mean field linear-quadratic-Gaussian systems with homogeneous population are retrieved, and, in addition, the results of this work are illustrated with a major and minor agent version of a game model of the synchronization of coupled nonlinear oscillators (see the Reference Subsection 5).

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